Technical Notes

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Velocity Predictions of Contraswirling Jets in a Suddenly Expanding Confinement

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Nomenclature

d_e = expansion diameter
d_i = inner jet diameter
d_o = outer jet outer diameter
k = turbulent kinetic energy
R = radial distance from axis of symmetry
R_o = outer jet outer radius ($=d_o/2$)
S = source term
U_i = inner jet average axial velocity, m/s
U_o = outer jet average axial velocity, m/s
U = axial velocity, m/s
W = circumferential velocity, m/s
X = axial distance from jet exit ($x/d_o=0.0$)
ϵ = energy dissipation rate
μ_t = eddy viscosity
$\mu_{\rm eff}$ = effective viscosity ($=\mu+\mu_t$)
ρ = density of the fluid
Γ = exchange coefficient

Introduction

MIXING of coaxial swirling jets in a confined space is very relevant to problems of combustion in double concentric-type burners. The relative direction of the swirl imparted to the two jets in a coaxial configuration has been found to affect, to a large degree, the mixing of the jets in the form of increased mass entrainment and the achievement of uniform flow in a shorter distance.

This Note presents the numerically predicted velocity characteristics of incompressible coaxial contraswirling jets exhausting into a sudden expanding confinement akin to a combustor. The geometry of the confinement chosen for the analysis was that of Singh[1] with a view to achieving a direct comparison with his experimental results. This geometry is shown in Fig. 1.

The elliptic forms of conservation equations of mass and axial, radial, and tangential momentum have been solved along with a two-equation turbulence model (k-ϵ) for a closure solution in a manner similar to the one adopted by Habib and Whitelaw[2] and Khalil.[3] They predicted flowfields for coaxial jets exhausting into a sudden expanding confinement with either the outer or central jet swirling. Habib and Whitelaw[2] observed a large discrepancy between their analytical and experimental results. They attributed this discrepancy to the presence of strong streamline curvatures that were not accounted in their analysis. A similar conclusion was also reached by Srinivasan and Mongia,[4] who predicted the flowfield in a cold model combustor with a confinement equivalent in diameter to the outer jet diameter for which the experimental results of Vu and Gouldin[5] were available for comparison. Using similar prediction technique, Ramos[6] has shown that, if the inlet conditions are properly selected, the k-ϵ model predicts a recirculation zone for both co- and contraswirling flows. All of his predictions were carried out in a model combustor with a confinement diameter equivalent to the outer jet outer diameter.

In the present analysis, results have been obtained by incorporating streamline curvature corrections into the ϵ equations of the turbulence model following the suggestions of Launder et al.[7]

Equations of Flow

In a cylindrical coordinate system, the basic laws for the present fluid flow problem can be expressed in the following general form[1]:

$$\frac{\partial}{\partial x}(\rho U\phi)+\frac{1}{r}\frac{\partial}{\partial r}(\rho rV\phi)-\frac{\partial}{\partial x}\left(\Gamma_\phi\frac{\partial\phi}{\partial x}\right)-\frac{1}{r}\frac{\partial}{\partial r}\left(r\Gamma_\phi\frac{\partial\phi}{\partial r}\right)-S_\phi=0 \tag{1}$$

where ϕ is a variable (representing 1, U, V, W, k, and ϵ for the conservation of mass, U momentum, V momentum, Wr momentum, turbulence energy, and dissipation rate, respectively), Γ_ϕ the exchange coefficient, and S_ϕ the source term. Both Γ_ϕ and S_ϕ are replaced with suitable terms corresponding to the form ϕ takes.

Models of Turbulence

The k-ϵ model of turbulence relates the eddy viscosity μ_t to the turbulent kinetic energy k and the dissipation rate ϵ by the expression

$$\mu_t=\rho C_\mu k^2/\epsilon \tag{2}$$

The transport equations for the turbulent kinetic energy k and the energy dissipation rate ϵ are obtained from Eq. (1) by replacing ϕ with k and ϵ, respectively, Γ_ϕ with $\mu_{\rm eff}/\sigma_k$, and $\mu_{\rm eff}/\sigma_\epsilon$, respectively, and S_ϕ with $(G-\rho\epsilon)$ and $(\epsilon/k)\cdot(C_1G-C_2\rho\epsilon)$, respectively. Here,

$$G=\mu_{\rm eff}\left\{2\left[\left(\frac{\partial U}{\partial x}\right)^2+\left(\frac{\partial V}{\partial r}\right)^2+\left(\frac{V}{r}\right)^2\right]+\left(\frac{\partial W}{\partial x}\right)^2+\left[r\frac{\partial}{\partial r}\left(\frac{W}{r}\right)\right]^2+\left[\frac{\partial U}{\partial r}+\frac{\partial V}{\partial x}\right]^2\right\}$$

and C_1, C_2, C_μ, σ_ϵ, and σ_k are constants of the model of turbulence.

The length scale constants C_1 and C_2 determine the ϵ equation. The effect of streamline curvature due to swirl has been

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accounted for by modifying the value of the constant C_2 on similar lines discussed by Launder, Priddin, and Sharma.[7] This correction is referred to as the LPS correction. Based on the gradient Richardson number R_i defined by

$$R_i = \frac{k^2}{\epsilon^2} \cdot \frac{W}{r^2} \cdot \frac{\partial (rW)}{\partial r}$$

the modified value C_2' of the constant C_2 can be expressed as

$$C_2' = C_2 f$$

where the function f is related to R_i by

$$f = 1 - 0.2R_i$$

Results and Discussion

Results have been obtained for the three cases A_1-A_3 outlined in Table 1. The velocity plots are presented in Figs. 2-4. The full continuous lines correspond to the predicted results and the dotted lines to the experimental results for different axial locations x/d_o. The experimental results obtained for the section at $x/d_o = 0$ formed the inlet conditions for the predictions.

Figure 2 shows the predicted velocity distribution for case A_1. It can be seen that the predicted axial velocity profile fairly well matches the experimentally obtained results both in magnitude and nature for most of the flow passage, except in the near-jet exit region. In this region, the centerline velocity is overpredicted and values in the outer shearing layer adjacent to the wall do not agree. These discrepancies are attributed mainly to the extreme velocity gradients present in these regions and the coarse 14×22 grids used for lack of computer space and time. In the near-jet exit region, the circumferential velocity distribution matching is good in the vicinity of the centerline. However, away from the centerline for $R/R_o > 0.6$, there is very little agreement. At $x/d_o = 2.75$, circumferential velocity matching throughout the section is good where both the experimental and predicted results show behavior of a forced vortex nature. At further downstream locations, the circumferential velocity component obtained from the experiments decays rapidly close to the boundary, whereas in the predicted distribution the forced vortex tendency still persists.

The results for case A_2 are shown in Fig. 3. Here, one observes that at $x/d_o = 1.25$ as the swirl intensity increases, the axial velocity matching of predictions with experiments is extremely good, but at downstream locations the matching of the predicted and experimental values deteriorates. This is because the axial velocity in the center is overpredicted, while close to the wall it is underpredicted. Prediction of the circumferential velocity has improved with complete matching at $x/d_o = 3.75$, where both experimental and predicted results show behavior similar to a forced vortex. Figure 4 shows the results of case A_3 for contraswirling jets exhausting into a confinement with an expansion ratio of 1.5. Such jets have matching similar to case A_2 at downstream locations. However, in the near-jet exit region, the centerline recirculation zone is not predicted.

A comparison of the results shows that, for case A_1, the predicted axial velocity at $x/d_o = 3.75$ matches within 5% of the experimental value for most of the radial section (except in the vicinity of the wall where it is about 20%). The deviation of circumferential velocity is below 5% in a small portion close to the symmetry axis and increases with the radius. With the increase in swirl intensity (case A_2), one observes fairly close agreement at $x/d_o = 1.25$ (below 10%), but it deteriorates downstream. However, the agreement of the circumferential velocities is nearly perfect at $x/d_o = 3.75$. Habib and Whitelaw[2] observed that the mismatch between the predicted and experimental results for nonswirling flow in a similar configuration was about 15%. This discrepancy increased further with the introduction of swirl in the outer jet. On quantitative comparison, one can see that introduction of the LPS correction reduces the discrepancy between the two axial velocity results to below 10% for the weak swirl combination in case A_1 and to about 20% for the higher swirl with improved matching for velocities not discussed by Habib and Whitelaw.[2]

Table 1 Test conditions

Case	Outer swirl S_o, deg[a]	Inner swirl S_i, deg	Velocity ratio U_o/U_i	Average velocity, m/s	Confinement expansion d_e/d_o
A_1	−30	15	1.6	42.0	2.0
A_2	−45	30	1.6	40.0	2.0
A_3	−45	30	1.6	44.0	1.5

[a]Vane angles in axial direction.

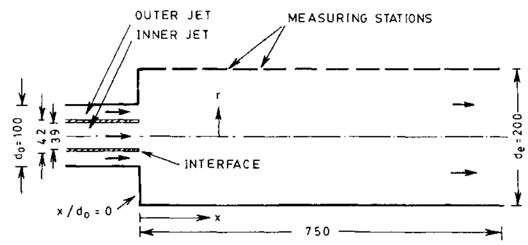
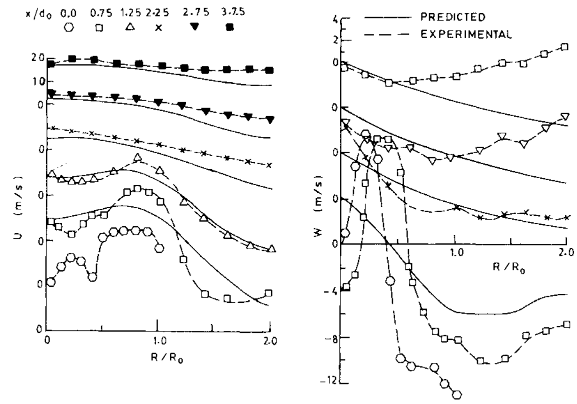
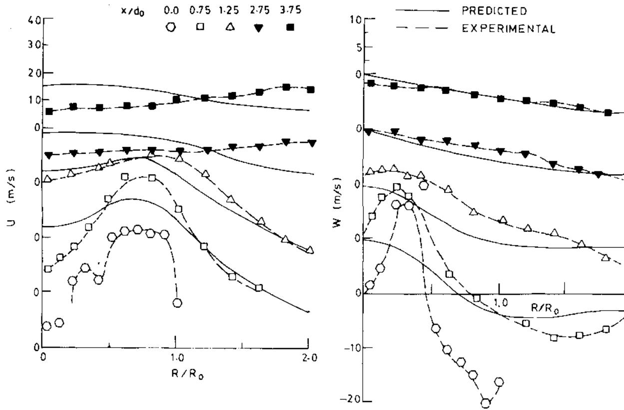


Fig. 1 Test section geometry (all dimensions in millimeters).



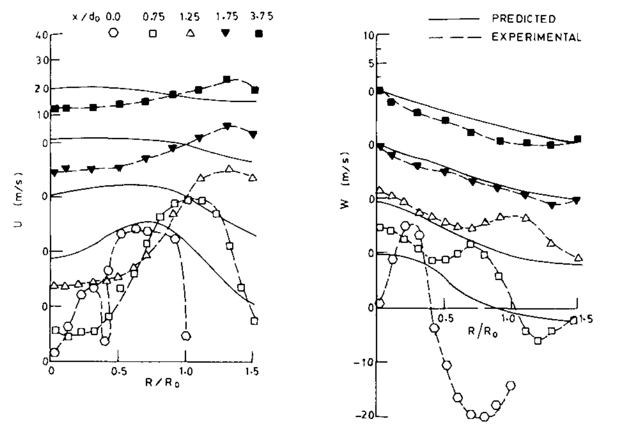
a) Axial velocity U. b) Circumferential velocity W.

Fig. 2 Comparison between experimental and predicted velocity profiles for case A_1.



a) Axial velocity U. b) Circumferential velocity W.

Fig. 3 Comparison between experimental and predicted velocity profiles for case A_2.



a) Axial velocity U. b) Circumferential velocity W.

Fig. 4 Comparison between experimental and predicted velocity profiles for case A_3.

Predictions for contra- and coswirling flows exhausting in a nonexpanding confinement made by Ramos and Sommer[8] matched only qualitatively. They noted that the introduction of streamline curvature correction might improve their results.

Conclusions

A close comparison of numerical predictions with experimental results for contraswirling jets in a suddenly expanding confinement leads to the following conclusions:

1) Theoretical predictions for the axial component of the velocity have closer agreement with the experimental values for weak swirls. The agreeement becomes poorer with the increase in the intensity of swirl.

2) Theoretical predictions for the circumferential component of velocity when compared with the experimental values show a reverse trend.

3) The LPS correction introduced to account for the streamline curvature improves the results of the prediction technique. The predictions are not satisfactory for high streamline curvatures caused by high swirl intensity and flow expansion.

4) The limitations of the turbulence model is also exposed in the zones having steep velocity gradients and in its inability to pick up the central recirculation zone (case A_3).

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Relationship Between Pseudocompressible and Unsteady Compressible Flow at Low Mach Numbers

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Nomenclature

a	= sound speed
L	= characteristic length scale
M	= Mach number = u/a
p	= pressure
Re	= Reynolds number = $\rho_\infty L u_R / \mu_R$
t	= time
T	= characteristic time scale
u	= velocity vector
u^*	= reference velocity used to form reference time
β	= "compressibility factor," used in method of pseudocompressibility
γ	= ratio of specific heats
δ	= boundary-layer thickness
θ	= temperature
ρ	= fluid density
τ	= stress tensor

Subscripts and Superscripts

R	= reference quantity
∞	= ambient value
$(\ )'$	= perturbation = $(\ ) - (\ )_\infty$

I. Introduction

THE method of pseudocompressibility[1] has been shown to work very well for the computation of steady viscous incompressible flows (e.g., Refs. 2 and 3). Rogers and Kwak[3] have also used this method for computing vortex shedding by a circular cylinder. They obtained good agreement between predicted streamlines and measurements. The question arises as to how well the method of pseudocompressibility can be expected in general to perform for unsteady-flow analyses, given its utilization of what is ostensibly a fictitious pressure time derivative. The answer to this question lies in the relationship between the differential equations used in this method to those of compressible flow. This relationship is derived below for "external" laminar flows.

II. Background

The differential equations used in the pseudocompressibility method are[3]

$$(1/\beta)p_t + \nabla \cdot u = 0 \tag{1a}$$

$$u_t + u \cdot \nabla u + \nabla p = \nu \nabla^2 u \tag{1b}$$

For flows that have a steady state, $p_t \to 0$ as $t \to \infty$, so that the above equations reduce to those of incompressible flow in the steady state.

In contrast, the equations of compressible flow are[4]

$$\rho_t + \nabla \cdot (\rho u) = 0 \tag{2a}$$

$$u_t + u \cdot \nabla u + \frac{1}{\rho}\nabla p - \frac{1}{\rho}\nabla \cdot \tau = 0 \tag{2b}$$

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